

Control chart for waiting time in system of M / M (a, b) / 1 Queueing model

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Abstract— Queueing models provide the basic frame work to get the optimum operating policies in production processes and service facilities. In transportation processes involving buses, aero planes, trains, ships, elevators and so on, bulk service queueing models are often encountered. Control chart technique analyzes the behavior of services and provides the prior idea about expected waiting time, maximum waiting time and minimum waiting time which in turn guarantees customer's satisfaction. Keeping this in view, the construction of control charts for waiting time in system of M / M (a, b) / 1 bulk service queueing model is proposed in this paper.

Index Terms— bulk service, control limits, exponential service, Poisson arrival, waiting time

1 INTRODUCTION

A queue or waiting line is formed when customers arriving for service, waiting for service if it is not immediate and if having waited for service, leaving the system after being served. Neuts(1967) introduced the general bulk service rule for single channel systems to deal with systems such as an elevator brings several people to another floor, a truck that transports several goods to various destinations etc. Queueing model consisting of bulk service has been discussed by Medhi (2008) and several authors. The general rule of bulk service model is that the service in batch takes a minimum size of 'a' units to a maximum size of 'b' units immediately after completion of a service if more than 'b' customers are waiting only the first 'b' customers are taken for service and the remaining will have to wait and if the number of customers is less than 'a' then the server remains idle until the batch size reaches 'a'.

Montgomery (2005) proposed a number of applications of control charts in assuring quality in manufacturing industries. Shore (2000) developed control chart by considering the first three moments for random queue length of M/M/1 queueing model. Khaparde and Dhabe (2010) constructed the control chart using the method of weighted variance for random queue length for M / M / 1 queueing model. Poongodi and Muthulakshmi (2013) analyzed waiting time of customers in system of M / M / 1 queueing model using the control chart technique. In this paper Shewhart control chart for waiting time in system of bulk service queueing model M / M (a, b) / 1 is presented.

2 MODEL DESCRIPTION

In M/M(a,b)/1 queueing system it is assumed that the arrivals occur at a single channel facility as Poisson process with the parameter λ . The customers are served on FCFS basis with mean service rate μ . There is no constraint on waiting capacity. The units are served in batches of size ranging from a to b, according to the general bulk service rule such that the service starts only when a minimum number of 'a' units present in the queue, the maximum service capacity is 'b'. The service times of the batches of sizes s ($a \leq s \leq b$) are assumed to have independent identical exponential distribution with the mean $1/\mu$. The service time distribution is assumed to be independent of the batch size s.

3 NOTATIONS

The state (0, n) denotes the service channel is idle and there are n units waiting in the queue where $0 \leq n \leq a-1$ and the state (1, n) denotes the service channel is busy and there are n units waiting in the queue where $n \geq 0$. Let

$P_{0,n}(t)$ = probability that at time t the system is in state (0, n)

and

$P_{1,n}(t)$ = probability that at time t the system is in state (1, n).

The steady state probability equations are

$$(\lambda + \mu) P_{1,n} = \lambda P_{1,n-1} + \mu P_{1,n+b}, \quad n = 1, 2, \dots \quad (1)$$

$$(\lambda + \mu) P_{1,0} = \lambda P_{0,a-1} + \mu \sum_{s=a}^b P_{1,s} \quad (2)$$

$$\lambda P_{0,0} = \mu P_{1,0} \quad (3)$$

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and

$$\lambda P_{0,n} = \lambda P_{0,n-1} + \mu P_{1,n}, \quad n = 1, 2, \dots, a-1, \quad a \neq 1 \quad (4)$$

The difference equation (1) can be written as

$$h(z) \{ P_{1,n} \} = 0, \quad n = 0, 1, 2, \dots \quad (5)$$

such that the characteristic equation is

$$h(z) \equiv \mu z^{b+1} - (\lambda + \mu) z + \lambda = 0 \quad (6)$$

Let $f(z) \equiv -(\lambda + \mu) z$ and $g(z) \equiv \mu z^{b+1} + \lambda$. Then $|g(z)| < |f(z)|$ for $|z| < 1$.

If $|g(z)| < |f(z)|$ then by Rouché's theorem $f(z)$ and $f(z) + g(z)$ will have the same number of zeros inside $|z| = 1$. Since

$f(z) \equiv -(\lambda + \mu) z$ has only one zero inside $|z| = 1$, there will be only one zero for $h(z) = f(z) + g(z)$ inside $|z| = 1$.

Denote this zero by r ($0 < r < 1$) and the other b roots by $r_1, r_2, r_3, \dots, r_b, |r_i| \geq 1$.

The solution of (1) can be written as

$$P_{1,n} = A r^n + \sum_{i=1}^b A_i r_i^n, \quad n = 0, 1, 2, \dots$$

Since $\sum_{n=0}^{\infty} P_{1,n} < 1$ and $|r_i| \geq 1$, A_i should be 0 for $i = 1, 2, \dots, b$.

$$\text{Hence } P_{1,n} = \frac{(1-r^b)}{1-r} r^{n+1} P_{0,0} \quad (7)$$

$$\text{and } P_{0,n} = \frac{1-r^{n+1}}{1-r} P_{0,0}, \quad n = 1, 2, \dots, a-1 \quad (8)$$

Using the normality condition we get

$$P_{0,0} = \left[\frac{a}{1-r} + \frac{r^{a+1} - r^{b+1}}{(1-r)^2} \right]^{-1} \quad (9)$$

Let $\rho = \lambda / b\mu$.

$$\text{From (6)} \quad \frac{\lambda}{\mu} = \frac{r(1-r^b)}{1-r}$$

Equations (7), (8) and (9) give the steady state probabilities of M/M (a,b)/1 queueing model.

4 WAITING TIME DISTRIBUTION

Let the random variable T denote the waiting time in the queue for an arriving unit in steady state and its pdf be $v(t)$.

Then for the model M/M (a, b) /1

$$v(t) = \left[\frac{\lambda P_{0,0}}{1-r} \right] \left[(1-r^b) e^{-\mu(1-r^b)t} + E(a-1, \lambda t) (1 - e^{-\mu(1-r^b)t}) \right],$$

$$0 < t < \infty,$$

$$\text{where } E(a-1, \lambda t) = e^{-(\lambda t)} \sum_{k=0}^{a-2} \frac{(\lambda t)^k}{k!}, \quad a \geq 2$$

5 MEAN AND VARIANCE

Mean = $E(T)$ =

$$\frac{\lambda P_{0,0}}{1-r} \left[\frac{1}{\mu^2 (1-r^b)} + \frac{a(a-1)}{2\lambda^2} + \frac{ar^{a+1}(1-r) - r^2(1-r^a)}{\lambda^2 (1-r)^2} \right] \quad (10)$$

and Variance = $\text{Var}(T) = E(T^2) - (E(T))^2 =$

$$\frac{\lambda P_{0,0}}{(1-r)} \left\{ \left[\frac{2}{\mu^3 (1-r^b)^2} + \frac{a(a-1)(a+1)}{3\lambda^3} + \frac{r^3}{\lambda^3 (1-r)^3} (2(1-r^a(1-a^2)) + ar^{a-1}(r^2(1-a) - (a+1))) \right] - \left[\frac{\lambda P_{0,0}}{(1-r)} \left[\frac{1}{\mu^2 (1-r^b)} + \frac{a(a-1)}{2\lambda^2} + \frac{ar^{a+1}(1-r) - r^2(1-r^a)}{\lambda^2 (1-r)^2} \right]^2 \right] \right\} \quad (11)$$

6 PARAMETERS OF CONTROL CHART

The parameters of Shewhart control chart, by approximating the statistic, the waiting time in the system, under consideration by a normal distribution, are given by

$$\left. \begin{aligned} \text{UCL} &= E(T) + 3 \sqrt{\text{Var}(T)} \\ \text{CL} &= E(T) \\ \text{LCL} &= E(T) - 3 \sqrt{\text{Var}(T)} \end{aligned} \right\} \quad (12)$$

The parameters of the control chart are found by using (10) and (11) in (12).

7 PARTICULAR CASES

7.1 Fixed size bulk service model M/M (k, k)/1

In M/M (a, b) /1 queueing model the batch size is considered as a constant with $a = b = k$.

Let P_n denote the steady state probability that the number in the system is n . Then

$$P_0 = P_{0,0} = \frac{1-r}{k},$$

$$P_n = P_{0,n} = \frac{1-r^{n+1}}{1-r} P_0 = \frac{1-r^{n+1}}{k}, \quad n = 1, 2, \dots, k-1$$

$$P_{n+k} = P_{1,n} = (\lambda/\mu) P_0 r^n = \rho (1-r) r^n, \quad n = 0, 1, 2, \dots$$

where $\rho = (\lambda/k\mu)$.

$$\text{Therefore } P_n = \begin{cases} \frac{1-r^{n+1}}{k}, & n = 1, 2, \dots, k-1 \\ Q(1-r)r^{n-k}\lambda, & n = k, k+1, \dots \end{cases}$$

The waiting time density is

$$v(t) = \mu Q \left[(1-r^k)e^{-\mu(1-r^k)t} + E(a-1, \lambda t)(1-e^{-\mu(1-r^k)t}) \right]$$

The mean and variance are

$$E(T) = \frac{1}{\lambda} \left(\frac{k-1}{2} + \frac{r^{k+1}}{1-r} \right) \quad \text{and}$$

$$\text{Var}(T) =$$

$$\frac{1}{\lambda^2} \left\{ \left(\frac{(k-1)(k+1)}{3} - \frac{r^{k+3}(2k+r(1-k)-r^{-1}(1+k))}{(1-r)^3} \right) - \left(\frac{k-1}{2} + \frac{r^{k+1}}{1-r} \right)^2 \right\}$$

7.2 M/M (1, b) /1 model

The steady state probabilities of M/M (a, b) /1 queueing model when a = 1 are

$$P_{0,0} = \frac{1-r}{1-r+(\lambda/\mu)} = \frac{(1-r)^2}{(1-r)^2+r(1-r^b)},$$

$$P_{1,n} = \frac{(1-r)(1-r^b)}{(1-r)^2+r(1-r^b)} r^{n+1}, \quad n = 0, 1, 2, \dots$$

The waiting time density is

$$v(t) = \left[\frac{\lambda P_{0,0}}{1-r} \right] \left[(1-r^b)e^{-\mu(1-r^b)t} \right]$$

and the expected waiting time and variance are

$$E(T) = \frac{r}{\mu[(1-r)^2+r(1-r^b)]}$$

$$\text{and Var}(T) = \left[\frac{2}{(1-r^b)} - \frac{r}{(1-r)^2+r(1-r^b)} \right]$$

7.3 M/M (a, ∞) /1 model

In M/M (a, b) /1 queueing model when the maximum service

capacity $b = \infty$, then $r = \frac{\lambda}{\lambda + \mu}$ and the steady state probabilities are obtained by putting $r^b = 0$ in equations (8) and (9).

$$P_{1,n} = \frac{1}{1-r} r^{n+1} P_{0,0}$$

$$P_{0,n} = \frac{1-r^{n+1}}{1-r} P_{0,0}, \quad n = 1, 2, \dots, a-1$$

$$P_{0,0} = \frac{(1-r)^2}{a(1-r) + r^{a+1}}$$

The waiting time density is

$$v(t) = \left[\frac{\lambda P_{0,0}}{1-r} \right] \left[e^{-\mu t} + E(a-1, \lambda t)(1-e^{-\mu t}) \right], \quad 0 < t < \infty,$$

$$\text{where } E(a-1, \lambda t) = e^{-(\lambda t)} \sum_{k=0}^{a-2} \frac{(\lambda t)^k}{k!}, \quad a \geq 2$$

The expected waiting time and the variance of waiting time are given by

$$E(T) = \frac{\lambda(1-r)}{a(1-r) + r^{a+1}} \left[\frac{a(a-1)}{2\lambda^2} + \frac{r^a(1+a(r^{-1}-1))}{\mu^2} \right]$$

and $\text{Var}(T) =$

$$\left\{ \left[\frac{a(a-1)(a+1)}{3\lambda^3} + \frac{\lambda(1-r)}{a(1-r) + r^{a+1}} \left[\frac{r^a}{\mu^3} (2(1-a^2) - ar^{-1}(r^2(1-a) - (a+1))) \right] \right] - \left[\frac{\lambda(1-r)}{a(1-r) + r^{a+1}} \left(\frac{a(a-1)}{2\lambda^2} + \frac{r^a(1+a(r^{-1}-1))}{\mu^2} \right) \right]^2 \right\}$$

8 NUMERICAL ANALYSIS

Assessment of waiting time in the system by means of control chart is carried out with numerical illustrations for selected values of a, b, λ and μ. The parameters of the control chart for various values of the arrival rates with constant service rate and various values of the service rates with constant arrival rate for different batch sizes of service are given respectively in Tables 1 and 2.

8.1 Table.1 Parameters of control chart for constant service rate $\mu=2$

λ	Batch sizes											
	a = 5 and b = 10			a = 5 and b = 20			a = 10 and b = 20			a = 10 and b = 10		
	q	CL	UCL	q	CL	UCL	q	CL	UCL	q	CL	UCL
1.00	0.050	2.002	7.998	0.025	1.997	7.994	0.025	4.500	15.212	0.050	4.500	15.212
1.50	0.075	1.339	5.332	0.038	1.338	5.332	0.038	3.000	10.142	0.075	3.000	10.141
2.00	0.100	1.013	4.008	0.050	1.009	4.004	0.050	2.250	7.607	0.100	2.251	7.606
2.50	0.125	0.823	3.229	0.063	0.825	3.230	0.063	1.801	6.085	0.125	1.801	6.085
3.00	0.150	0.702	2.732	0.075	0.699	2.728	0.075	1.502	5.072	0.150	1.503	5.073
3.50	0.175	0.621	2.404	0.088	0.618	2.396	0.088	1.290	4.349	0.175	1.291	4.353
4.00	0.200	0.566	2.186	0.100	0.560	2.170	0.100	1.132	3.811	0.200	1.134	3.817
4.50	0.225	0.529	2.046	0.113	0.519	2.017	0.113	1.010	3.398	0.225	1.014	3.406
5.00	0.250	0.504	1.961	0.125	0.491	1.917	0.125	0.914	3.072	0.250	0.920	3.084
5.50	0.275	0.488	1.918	0.138	0.476	1.856	0.138	0.837	2.811	0.275	0.845	2.831
6.00	0.300	0.479	1.905	0.150	0.462	1.817	0.150	0.774	2.601	0.300	0.786	2.631
6.50	0.325	0.475	1.917	0.163	0.448	1.790	0.163	0.722	2.430	0.325	0.738	2.475
7.00	0.350	0.477	1.947	0.175	0.443	1.783	0.175	0.680	2.289	0.350	0.700	2.357
7.50	0.375	0.483	1.993	0.188	0.437	1.781	0.188	0.644	2.177	0.375	0.669	2.272
8.00	0.400	0.512	2.052	0.200	0.429	1.780	0.200	0.614	2.087	0.400	0.646	2.216
8.50	0.425	0.516	2.122	0.213	0.434	1.796	0.213	0.589	2.014	0.425	0.630	2.187
9.00	0.450	0.520	2.203	0.225	0.436	1.813	0.225	0.568	1.958	0.450	0.619	2.183
9.50	0.475	0.539	2.295	0.238	0.437	1.828	0.238	0.550	1.915	0.475	0.613	2.204
10.00	0.500	0.561	2.399	0.250	0.435	1.841	0.250	0.536	1.882	0.500	0.612	2.247
10.50	0.525	0.587	2.515	0.263	0.450	1.876	0.263	0.524	1.860	0.525	0.617	2.314
11.00	0.550	0.617	2.644	0.275	0.443	1.884	0.275	0.514	1.846	0.550	0.627	2.404
11.50	0.575	0.651	2.790	0.288	0.455	1.917	0.288	0.506	1.840	0.575	0.643	2.518
12.00	0.600	0.690	2.953	0.300	0.467	1.948	0.300	0.500	1.840	0.600	0.664	2.656
12.50	0.625	0.734	3.139	0.313	0.477	1.979	0.313	0.496	1.844	0.625	0.693	2.824
13.00	0.650	0.786	3.351	0.325	0.455	1.971	0.325	0.493	1.854	0.650	0.729	3.022
13.50	0.675	0.846	3.596	0.338	0.493	2.036	0.338	0.491	1.869	0.675	0.775	3.259
14.00	0.700	0.915	3.877	0.350	0.497	2.062	0.350	0.491	1.886	0.700	0.831	3.535
14.50	0.725	0.999	4.215	0.363	0.499	2.085	0.363	0.491	1.909	0.725	0.902	3.873
15.00	0.750	1.098	4.614	0.375	0.499	2.106	0.375	0.493	1.933	0.750	0.989	4.274
15.50	0.775	1.221	5.109	0.388	0.494	2.122	0.388	0.496	1.960	0.775	1.101	4.774
16.00	0.800	1.374	5.721	0.400	0.534	2.194	0.400	0.499	1.991	0.800	1.243	5.393
16.50	0.825	1.570	6.507	0.413	0.525	2.208	0.413	0.504	2.023	0.825	1.429	6.189
17.00	0.850	1.831	7.550	0.425	0.511	2.215	0.425	0.509	2.060	0.850	1.680	7.245
17.50	0.875	2.201	9.033	0.438	0.553	2.292	0.438	0.515	2.098	0.875	2.041	8.741
18.00	0.900	2.743	11.198	0.450	0.532	2.295	0.450	0.522	2.139	0.900	2.574	10.921
18.50	0.925	3.672	14.916	0.463	0.575	2.376	0.463	0.530	2.182	0.925	3.495	14.656
19.00	0.950	5.526	22.334	0.475	0.545	2.368	0.475	0.539	2.227	0.950	5.341	22.092
19.50	0.975	11.019	44.305	0.488	0.590	2.455	0.488	0.548	2.276	0.975	10.827	44.084

8.2 Table.2 Parameters of control chart for constant arrival rate $\lambda = 7$

μ	Batch sizes											
	a = 5 and b = 10			a = 5 and b = 20			a = 10 and b = 20			a = 10 and b = 10		
	q	CL	UCL	q	CL	UCL	q	CL	UCL	q	CL	UCL
1.00	0.700	1.830	7.753	0.350	0.967	4.091	0.350	0.981	3.773	0.700	1.662	7.070
1.25	0.560	1.008	4.320	0.280	0.721	3.040	0.280	0.817	2.948	0.560	1.012	3.913
1.50	0.467	0.710	3.017	0.233	0.583	2.432	0.233	0.741	2.570	0.467	0.819	2.925
1.75	0.400	0.562	2.345	0.200	0.498	2.043	0.200	0.702	2.385	0.400	0.739	2.532
2.00	0.350	0.477	1.947	0.175	0.442	1.782	0.175	0.680	2.290	0.350	0.700	2.357
2.25	0.311	0.424	1.696	0.156	0.403	1.602	0.156	0.667	2.240	0.311	0.678	2.273
2.50	0.280	0.388	1.530	0.140	0.376	1.475	0.140	0.659	2.212	0.280	0.666	2.230
2.75	0.255	0.363	1.419	0.127	0.356	1.385	0.127	0.654	2.196	0.255	0.658	2.207
3.00	0.233	0.346	1.341	0.117	0.341	1.321	0.117	0.650	2.187	0.233	0.654	2.193
3.25	0.215	0.333	1.288	0.108	0.330	1.275	0.108	0.648	2.182	0.215	0.650	2.186
3.50	0.200	0.324	1.249	0.100	0.322	1.241	0.100	0.647	2.178	0.200	0.648	2.181
3.75	0.187	0.316	1.222	0.093	0.315	1.217	0.093	0.646	2.176	0.187	0.647	2.178
4.00	0.175	0.311	1.202	0.088	0.310	1.199	0.088	0.645	2.175	0.175	0.646	2.176
4.25	0.165	0.303	1.187	0.082	0.306	1.185	0.082	0.644	2.174	0.165	0.645	2.175
4.50	0.156	0.303	1.177	0.078	0.302	1.175	0.078	0.644	2.174	0.156	0.644	2.175
4.75	0.147	0.300	1.169	0.074	0.300	1.167	0.074	0.644	2.174	0.147	0.644	2.174
5.00	0.140	0.298	1.163	0.070	0.297	1.162	0.070	0.643	2.173	0.140	0.644	2.174
5.25	0.133	0.296	1.158	0.067	0.296	1.157	0.067	0.643	2.173	0.133	0.644	2.174
5.50	0.127	0.294	1.154	0.064	0.294	1.154	0.064	0.643	2.173	0.127	0.643	2.173
5.75	0.122	0.293	1.152	0.061	0.293	1.151	0.061	0.643	2.173	0.122	0.643	2.173
6.00	0.117	0.292	1.150	0.058	0.292	1.149	0.058	0.643	2.173	0.117	0.643	2.173
6.25	0.112	0.291	1.148	0.056	0.291	1.148	0.056	0.643	2.173	0.112	0.643	2.173
6.50	0.108	0.290	1.147	0.054	0.290	1.147	0.054	0.643	2.173	0.108	0.643	2.173
6.75	0.104	0.290	1.146	0.052	0.290	1.146	0.052	0.643	2.173	0.104	0.643	2.173
7.00	0.100	0.289	1.145	0.050	0.289	1.145	0.050	0.643	2.173	0.100	0.643	2.173
7.25	0.097	0.289	1.145	0.048	0.289	1.144	0.048	0.643	2.173	0.097	0.643	2.173
7.50	0.093	0.288	1.144	0.047	0.288	1.144	0.047	0.643	2.173	0.093	0.643	2.173
7.75	0.090	0.288	1.144	0.045	0.288	1.144	0.045	0.643	2.173	0.090	0.643	2.173
8.00	0.088	0.288	1.143	0.044	0.288	1.143	0.044	0.643	2.173	0.088	0.643	2.173
8.25	0.085	0.288	1.143	0.042	0.288	1.143	0.042	0.643	2.173	0.085	0.643	2.173
8.50	0.082	0.287	1.143	0.041	0.287	1.143	0.041	0.643	2.173	0.082	0.643	2.173
8.75	0.080	0.287	1.143	0.040	0.287	1.143	0.040	0.643	2.173	0.080	0.643	2.173
9.00	0.078	0.287	1.143	0.039	0.287	1.143	0.039	0.643	2.173	0.078	0.643	2.173
9.25	0.076	0.287	1.143	0.038	0.287	1.143	0.038	0.643	2.173	0.076	0.643	2.173
9.50	0.074	0.287	1.143	0.037	0.287	1.143	0.037	0.643	2.173	0.074	0.643	2.173
9.75	0.072	0.287	1.143	0.036	0.287	1.143	0.036	0.643	2.173	0.072	0.643	2.173
10.00	0.070	0.287	1.143	0.035	0.287	1.143	0.035	0.643	2.173	0.070	0.643	2.173

9 CONCLUSION

The numerical values in Table.1 reveal the effect of different arrival rates with constant service rate on the traffic intensity and the parameters of the control chart.

- i. For fixed λ and a , increase in b decreases the value of the traffic intensity, CL and UCL.
- ii. For fixed λ and b , increase in ' a ' keeps the traffic intensity constant and increases CL and UCL.
- iii. For fixed μ , increase in λ increases the traffic intensity irrespective of the batch size.
- iv. For fixed μ , increase in λ decreases CL and UCL to a certain level and then increases irrespective of the batch size.

The numerical values in Table.2 yield the effect of different service rates with constant arrival rate on the traffic intensity and the parameters of the control chart.

- i. For fixed μ and a , increase in b decreases the value of the traffic intensity, CL and UCL.
- ii. For fixed values of μ and b , increase in ' a ' keeps the traffic intensity constant and increases CL and UCL.
- iii. For fixed λ , increase in μ decreases the traffic intensity, CL and UCL.

Numerical illustrations with varying arrival rates and constant service rate show that there exists an optimal value for the parameters of the control chart. Increase in the maximum limit of the batch size decreases the traffic intensity.

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